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The theory of electromagnetic wave propagation in magnetic multilayers

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Abstract. This paper presents a theory of electromagnetic (EM) wave propagation through magnetic multilayers and superlattices based on the propagation matrix **P** of a magnetic film. By using this matrix **P**, we obtained the transmission and reflection coefficients of layered magnetic media, including semi-infinite magnetic surfaces, magnetic multilayers and semi-infinite magnetic superlattices. The numerical results show that the EM modes of a magnetic layer system are excited and manifested as the sharp dips in the s-polarized reflection, and the dispersion curves of magnetic polaritons can be measured by a method similar to the attenuated total reflection technique. The analytic dispersion relations of magnetic polaritons for arbitrary magnetic multilayer systems are represented by the matrix **P**.

1. Introduction

New techniques have been developed in recent years to produce magnetic layered systems. Multilayer stackings of different magnetic materials with very small layer thicknesses and discontinuous magnetization profiles intercalated with dielectric layers are obtainable. Rapid progress in such techniques as molecular beam epitaxy or organometallic chemical vapour deposition enables us to grow systems with pre-determined film thickness and with sharp interfaces. Such systems seem to provide a new type of material which does not exist naturally. One can create materials with properties distinct from those of any single constituent. The artificially layered systems with imposed superperiodic structure are called superlattices. Some features of the layered structures can be explained in terms of modified single-film properties.

In most of the experimental and theoretical work on artificially magnetic layered systems, researchers have dealt with a study of the excitations composed of two different materials. Excitations at a long-wavelength limit (dipolar magnetostatic modes) in superlattices consisting of alternating magnetic and non-magnetic layers have been considered theoretically and experimentally [1–3]. It has been shown that among other features the surface magnetostatic waves (Damon–Easbach modes) of the magnetic layers interact across the non-magnetic films via the dipole field generated by the spin motion and consequently form a band of collective bulk excitations of the superlattice. Further modification of the spin-wave dispersion can be realized if the spin waves couple to electromagnetic (EM) waves to form so-called polaritons [4, 5]. More recently, Barnas [6, 7] obtained the bulk and surface magnetostatic dispersion relations for multiple-type infinite and semi-infinite magnetic superlattices by the transfer matrix method.

All the above studies, except those by Damon and Easbach [8] and by Gruberg [9], which treat single and double films, discuss infinite and semi-infinite magnetic superlattices. Vecris and Quinn [10] have mentioned this point and have recently presented a diagrammatic approach for obtaining the exact dispersion relations for finite magnetic superlattices. Yet their method is difficult to follow in practical computations and designs. For example, for a six-layer system, the dispersion contains $2^6 = 64$ terms with each term being a complicated expression.

On the other hand, as we know, in magnetic multilayers, the solutions of Maxwell's equations at the vacuum–magnet interface are of two general types: firstly, scattering solutions obeying the law of reflection and secondly, EM surface waves or magnetic surface polaritons. Both of these types of solution are states of constant wavevector component parallel to the surface and represent distinct and independent solutions for the EM modes of the system. Yet the former data, e.g. the transmission and reflection of EM waves through magnetic multilayers, have not received as much attention as spin-wave investigations. The non-reciprocal reflection of EM waves at a magnetic surface is one of the basic characteristics. Non-reciprocal devices are made according to this principle. The non-reciprocal reflection of semi-infinite ferromagnets and semi-infinite antiferromagnets has been investigated by Srivastava [11] and Stamps *et al* [12], respectively. Very recently, the lateral displacement of a light beam at a ferrite interface was calculated [13]. There is a need for a complete theory of EM wave propagation through magnetic multilayers and magnetic superlattices. In this work, we present such a theory based on a propagation matrix **P**. In order to simplify the presentation, we assume that the polarization of EM waves is parallel to the static magnetization and both are perpendicular to the incidence plane. The numerical calculations for a single magnetic layer and a double magnetic layer show that the magnetic surface polaritons were excited and manifested by sharp dips in the s-polarized reflectance, which is similar to the well known attenuated total reflection (ATR) technique discovered by Otto, Raether and Kretschmann. They pointed out that bound EM surface modes on condensed matter such as metals could be generated using the ATR method [14]. The dispersion curves of magnetic polaritons can be measured by a method similar to the ATR technique. Based on the matrix **P**, the dispersion relations of magnetic polaritons are expressed by a 2×2 matrix for any magnetic layer system.

2. Theory

2.1. *Effective conductance*

A uniform plane wave of frequency ω, initially propagating in a lossless dielectric medium (air) is incident on the dielectric–planar ferrite interface at $x = 0$. The electric vector of the incident wave is assumed to be parallel to the static magnetization, which in turn is normal to the plane of the incidence. As shown in figure 1, the magnetic layers are arbitrarily assumed. The $x < 0$ space is filled with non-magnetic dielectric medium. Then

$$\begin{aligned} E_Z &= A_I \exp\{i[\omega t - k_d \sin(q_i) y - k_d \cos(q_i) x]\} \\ H_Y^+ &= -i(c/w)(\partial E_z/\partial X) = -N_d \cos(q_i)\, E_Z \qquad (x < 0) \end{aligned} \tag{1}$$

while, for the reflected wave propagating along the negative x direction, we have

$$H_Y^- = N_d \cos(q_i)\, E_Z. \tag{2}$$

It is defined that

$$Y^{-} = H_Y^{-}/E_Z \qquad \text{or} \qquad Y^{+} = -H_Y^{+}/E_Z \tag{3}$$

for a non-magnetic medium. We have $Y^- = Y^+ = Y$. Y is the effective conductance of the non-magnetic dielectric medium, which is the typical method used in thin-film optics [15].

For a magnetic medium, the magnetic susceptibility is a tensor, which is given by

$$\mathbf{U} = \begin{bmatrix} u_1 & \mathrm{i}u_2 & 0 \\ -\mathrm{i}u_2 & u_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{4}$$

The magnetic susceptibility is frequency dependent and can be written, for the long-wavelength limit, as

$$u_1 = 1 + 4\pi\gamma^2 H_0 M_0/(\gamma^2 H_0^2 - \omega^2) \qquad u_2 = 4\pi\gamma\omega M_0/(\gamma^2 H_0^2 - \omega^2) \tag{5}$$

for a ferromagnetic material.

For an EM wave propagating along the positive x direction,

$$\begin{aligned} E_z^{\mathrm{f}} &= A_{\mathrm{T}} \exp\{\mathrm{i}[\omega t - k_{\mathrm{f}} \sin(q_{\mathrm{t}}) y - k_{\mathrm{f}} \cos(q_{\mathrm{t}}) x]\} \\ H_y^{+\mathrm{f}} &= (\mathrm{i}c_0/\omega u_{\mathrm{eff}})[(\mathrm{i}u_2/u_1)(\partial E_z^{\mathrm{f}}/\partial y) - \partial E_z^{\mathrm{f}}/\partial x] \\ &= (N_{\mathrm{f}}/\sqrt{u_{\mathrm{eff}}})\{[-\mathrm{i}u_2 \sin(q_{\mathrm{t}})]/u_1 + \cos(q_{\mathrm{t}})\} E_z^{\mathrm{f}} \end{aligned} \tag{6}$$

where $k_{\mathrm{f}} = u_{\mathrm{eff}}^{1/2} N_{\mathrm{f}} k_0$, $u_{\mathrm{eff}} = (u_1^2 - u_2^2)/u_1$ and $N_{\mathrm{f}} = \epsilon_{\mathrm{f}}^{1/2}$. N_{f} is frequency independent.

For a negative propagating wave we have

$$H_y^{-\mathrm{f}} = (N_{\mathrm{f}}/\sqrt{u_{\mathrm{eff}}})\{[\mathrm{i}u_2 \sin(q_{\mathrm{t}})]/u_1 + \cos(q_{\mathrm{t}})\} E_z^{\mathrm{f}}. \tag{7}$$

We define

$$Y^{-} = H_y^{-\mathrm{f}}/E_z = \{[N_{\mathrm{f}} \cos(q_{\mathrm{t}})]/\sqrt{u_{\mathrm{eff}}}\}\{[\mathrm{i}u_2 \tan(q_{\mathrm{t}})]/u_1 + 1\} \tag{8}$$

$$Y^{+} = -H_y^{+\mathrm{f}}/E_z = \{[N_{\mathrm{f}} \cos(q_{\mathrm{t}})]/\sqrt{u_{\mathrm{eff}}}\}\{[-\mathrm{i}u_2 \tan(q_{\mathrm{t}})]/u_1 + 1\}. \tag{9}$$

In the general case for magnetic materials, it is found that $Y^- \neq Y^+$, which differs from the non-magnetic medium. It is this difference that results in the non-reciprocal reflectance of semi-infinite magnetic materials.

2.2. *The propagation matrix of a magnetic film*

Now we further our investigation by considering a magnetic film, as shown in figure 2. At the first interface, we have

$$E_0^+ + E_0^- = E_{11}^- + E_{12}^+ \qquad Y_0(E_0^- - E_0^+) = Y_1^- E_{11}^- - Y_1^+ E_{12}^+. \tag{10}$$

From equation (10), the following equation is derived:

$$\begin{bmatrix} E_{11}^- \\ E_{12}^+ \end{bmatrix} = \frac{1}{Y_1^+ + Y_1} \begin{bmatrix} Y_1^+ & 1 \\ Y_1^- & -1 \end{bmatrix} \begin{bmatrix} E_0 \\ H_0 \end{bmatrix} \tag{11}$$

where E_0 and H_0 are the electric field and magnetic field, respectively, at the first interface. At the other interface of the film, we have

$$E_2^- = E_{21}^+ + E_{22}^- \qquad Y_0 E_2^- = -Y_1^+ E_{21}^+ + Y_1^- E_{22}^-. \tag{12}$$

Considering the propagation phase factor, we have

$$\begin{bmatrix} E_2^- \\ Y_0 E_2^- \end{bmatrix} = \frac{1}{Y_1^+ + Y_1} \begin{bmatrix} \exp(\mathrm{i}\delta) & \exp(-\mathrm{i}\delta) \\ \exp(\mathrm{i}\delta) Y_1^- & -\exp(-\mathrm{i}\delta) Y_1^+ \end{bmatrix} \begin{bmatrix} Y_1^+ & 1 \\ Y_1^- & -1 \end{bmatrix} \begin{bmatrix} E_0 \\ H_0 \end{bmatrix} \tag{13}$$

where

$\delta = [2\pi k_f \cos(q_t)\, d_0]/\lambda k_0 = [2\pi \sqrt{u_{\mathrm{eff}}} N_f \cos(q_t)\, d_0]/\lambda$.

We define the propagation matrix of a magnetic film as

$$\begin{aligned} \mathbf{P} &= \frac{1}{Y_1^+ + Y_1^-} \begin{bmatrix} \exp(\mathrm{i}\delta) & \exp(-\mathrm{i}\delta) \\ \exp(\mathrm{i}\delta) Y_1^- & -\exp(-\mathrm{i}\delta) Y_1^+ \end{bmatrix} \begin{bmatrix} Y_1^+ & 1 \\ Y_1^- & -1 \end{bmatrix} \\ &= \begin{bmatrix} \cos(\delta) - (\mathrm{i}M/L)\sin(\delta) & (\mathrm{i}/L)\sin(\delta) \\ [\mathrm{i}(L^2 - M^2)/L]\sin(\delta) & \cos(\delta) + (\mathrm{i}M/L)\sin(\delta) \end{bmatrix} \end{aligned} \tag{14}$$

where

$$\begin{aligned} L &= (N_f/\sqrt{u_{\mathrm{eff}}})\cos(q_t) = (N_f/\sqrt{u_{\mathrm{eff}}})[1 - \sin^2(q_i)/u_{\mathrm{eff}} N_f^2]^{1/2} \\ M &= [\mathrm{i} N_f u_2 \sin(q_t)]/\sqrt{u_{\mathrm{eff}}} u_1 = [\mathrm{i} u_2 \sin(q_i)]/u_{\mathrm{eff}} u_1 \\ \delta &= (2\pi \sqrt{u_{\mathrm{eff}}} N_f d_0/\lambda)[1 - \sin^2(q_i)/u_{\mathrm{eff}} N_f^2]^{1/2}. \end{aligned} \tag{15}$$

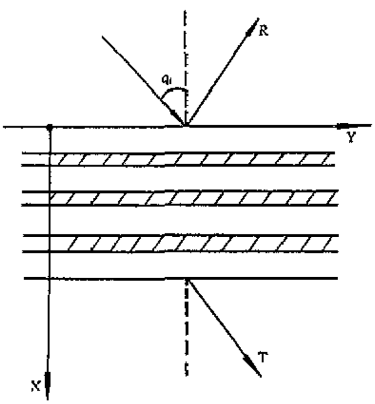

Figure 1. The geometry of the problem.

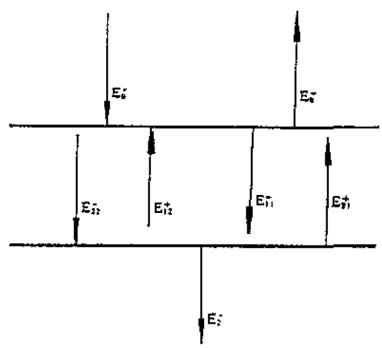

Figure 2. The diagram of EM propagation in a magnetic film.

From the expression for the matrix **P**, we know that **P** is a **U** matrix, i.e. $|\mathbf{P}| = 1$. For a non-magnetic film, $M_0 = 0$, $M = 0$ and $u_{\mathrm{eff}} = 1$. The matrix **P** has the form

$$\mathbf{P} = \begin{bmatrix} \cos(\delta) & [\mathrm{i}\sin(\delta)]/Y \\ \mathrm{i}Y\sin(\delta) & \cos(\delta) \end{bmatrix}. \tag{16}$$

2.3. *The propagation matrix of multilayers*

The matrix **P** of magnetic multilayers can be derived as

$$\mathbf{P} = \prod_{i=1}^{N} \mathbf{P}_i = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \tag{17}$$

where N is the total layer number.

2.4. *Transmission and reflection of waves through magnetic multilayers*

Assume that the conductance of the dielectric medium where the waves finally propagate is Y_d. Set

$$\begin{bmatrix} B \\ C \end{bmatrix} = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \begin{bmatrix} 1 \\ Y_d \end{bmatrix}. \tag{18}$$

From [15], the effective conductance of the multilayer system is

$$Y_{eff} = C/B = (P_{21} + Y_d P_{22})/(P_{11} + Y_d P_{12}). \tag{19}$$

Therefore

$$\begin{aligned} R &= [N\cos(q_i) - Y_{eff}]/[N\cos(q_i) + Y_{eff}] \\ T &= [2N\cos(q_i)\, Y_{eff}]/[N\cos(q_i) + Y_{eff}] \end{aligned} \tag{20}$$

where $N\cos(q_i) = Y_0$ is the conductance of the dielectric medium in which the EM wave initially propagates. Y_{eff} is given by equations (8) and (9) for semi-infinite magnetic materials and by equation (19) for magnetic films and multilayers.

2.5. *Semi-infinite magnetic superlattice*

The propagation matrix of a periodic unit containing N layers is $\mathbf{P}_u = \Pi_{i=1}^{N} \mathbf{P}_i$. From the form of the matrix $\mathbf{P}$, we know that $|\mathbf{P}_i| = 1, |\mathbf{P}_u| = 1$. The eigenvalue problem for the matrix $\mathbf{P}$ has the form

$$\mathbf{P}_u G_\pm = \lambda_\pm G_\pm \tag{21}$$

where $\lambda_\pm$ are the two eigenvalues and $G_\pm$ are the corresponding eigenvectors. In general, two different situations are possible.

(i) The eigenvalues λ_+ and λ_- are both complex with $\lambda_+ = \lambda_-^*$ and $|\lambda_\pm| = 1$.
(ii) The eigenvalues λ_+ and λ_- are both real and fulfil the condition $\lambda_+\lambda_- = 1$.

In both case (i) and case (ii), the following relation is fulfilled:

$$\lambda_+ + \lambda_- = P_{11} + P_{22}. \tag{22}$$

The eigenvectors corresponding to λ_+ and λ_- can be written, in the general case ($P_{12} \neq 0$), in the form

$$G_\pm = \frac{1}{Z_\pm} \begin{bmatrix} 1 \\ (\lambda_\pm - P_{11})/P_{12} \end{bmatrix} \tag{23}$$

where $Z_\pm$ are the normalizing factors: $Z_\pm = 1 + |\lambda_\pm - P_{11}|^2/|P_{12}|^2$.

At frequencies for which the matrix $\mathbf{P}_u$ is diagonal ($P_{12} = P_{21} = 0$), the eigenvalues are equal to P_{11} and P_{22} and the eigenvectors are of the form $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$. The complex eigenvalues of the matrix $\mathbf{P}_u$ correspond to the propagation solutions in the magnetic superlattice. According to Bloch's theorem, we write these eigenvalues in the form $\lambda_\pm = \exp(\pm i\beta D)$, where D is the thickness of a periodic unit. Therefore from equation (22) we have

$$\cos(\beta D) = \tfrac{1}{2}(P_{11} + P_{22}). \tag{24}$$

Of course, only $\lambda_- = \exp(-\mathrm{i}\beta D)$ (or equivalently $\lambda_+ = \exp(+\mathrm{i}\beta D)$) may be involved in solutions for propagation along the positive (or negative) x axis.

The following relation is valid for propagation solutions:

$$\begin{bmatrix} E_1 \\ -Y_{\mathrm{eff}}E_1 \end{bmatrix} = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \begin{bmatrix} E_0 \\ H_0 \end{bmatrix} = \frac{1}{Z_-} \begin{bmatrix} 1 \\ [\exp(-\mathrm{i}\beta D) - P_{11}]/P_{12} \end{bmatrix}. \tag{25}$$

Therefore

$$Y_{\mathrm{eff}} = [P_{11} - \exp(-\mathrm{i}\beta D)]/P_{12} \tag{26}$$

for the propagation solution. We also have

$$Y_{\mathrm{eff}} = [P_{11} - \exp(-\alpha D)]/P_{12} \tag{27}$$

for decaying solutions. The latter solution corresponds to case (ii) of the real eigenvalue for the matrix **P** with

$$\cosh(\alpha D) = \tfrac{1}{2}(P_{11} + P_{22}). \tag{28}$$

Whether equation (27) or (26) should be used is determined by the value of $\frac{1}{2}(P_{11} + P_{22})$. If it is larger than 1, equation (27) should be used; otherwise equation (26) should be employed. From equation (20), the reflection of waves through a semi-infinite magnetic superlattice can be obtained.

3. Numerical calculations

From equation (20), the reflection coefficient can be computed when Y_{eff} is obtained for any layer system. From equation (15), we know that total reflection occurs in the case when δ is imaginary. The total reflection of EM wave propagation across the ferrite interface was discussed by Srivastava [16]. The condition for the occurrence of OR/TR for a ferromagnetic film can be written from equation (15) as

$$u_{\mathrm{eff}} - \sin^2(q_{\mathrm{i}})/N_{\mathrm{f}}^2 \gtrless 0. \tag{29}$$

When the microwave undergoes total internal reflection at a dielectric (or air)–ferrite interface, an evanescent field penetrates roughly an optical wavelength into the ferrite. When the thickness of the ferrite is equal to or smaller than the wavelength of the wave, the evanescent field in the ferrite couples the surface magnetic polaritons. If the parallel component of the wavevector coincides with the propagation vector of an EM mode of oscillation of the film system, then a resonant absorption can occur. For a ferrite layer system, the EM mode of the oscillation (magnetic polariton) is related to the biasing field and is non-reciprocal. We have written a FORTRAN program to compute any ferrite layer system irrespective of whether δ in equation (15) is real or imaginary. Figure 3 shows the relations of the reflection coefficient R_+, R_- and $|\delta_+ - \delta_-|$ versus the biasing field for an asymmetric single layer. The parameters are $f = 90$ GHz, $4\pi M_0 = 3$ kOe, $n_{\mathrm{f}} = 3.16$, $q_{\mathrm{i}} = 45°$, $n_{\mathrm{d}} = 1 \pm 1.41\mathrm{i}$, $n_0 = 1$ and $d = 10$ μm. From the figure, we see that R_+ and R_- are different, i.e. the reflection of the EM microwave from the magnetized ferrimagnetic film is phase-wise and amplitude-wise non-reciprocal for an asymmetric layer system. There exists a dip in each reflection curve which occurs because of the excitation of a magnetic polariton propagating along the film surface. The power of incident microwaves is fed into the magnetic polariton. In this case the wavevector parallel to the surface matches that of the magnetic polariton at the air–M surface or dielectric–M interface. The parallel component of the wavevector of the incident wave can be varied in two ways: one is to fix the wavelength and to vary the angle of incidence, and the other is to fix the angle of incidence and to vary the wavelength, as widely used in the ATR technique [5]. For a given parallel component of the wavevector of the incident wave, the wavevector of the magnetic polariton is varied by changing the biasing field. For an asymmetric single layer, there are two different dispersion curves corresponding to positive and negative propagation. Thus the dip in R_+ differs from that in R_-. Figure 3(*b*) shows the differential phase shift of R_+ and R_- versus H, which is large near the resonant range. Figure 4 shows the result for a double magnetic film which has two dips corresponding to the two dispersion curves of the double magnetic film [9]. Figure 5 shows the influence of the distance between the two magnetic films, where the full curve corresponds to $d_0 = 0$ and the broken curve to $d_0 = 100$ μm. The results agree with those of Gruberg [9]. Our results show that the dispersion relations of the EM mode of the magnetic layer system can be demonstrated by EM microwave oblique incidence and ATR.

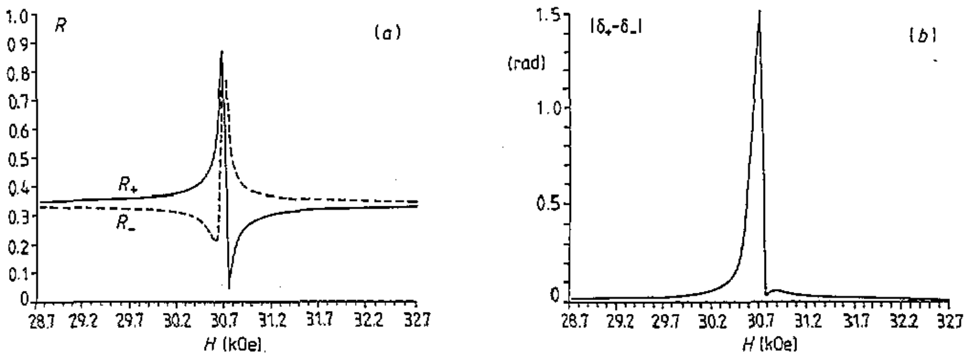

Figure 3. Variations in (*a*) the forward and reverse reflection coefficients and (*b*) the differential phase shift with biasing field for an asymmetric single layer. The parameters are $f = 90$ GHz, $4\pi M_0 = 3$ kOe, $n_f = 3.16$, $q_i = 45°$, $n_d = 1 \pm 1.41i$, $n_0 = 1$ and $d = 10$ μm.

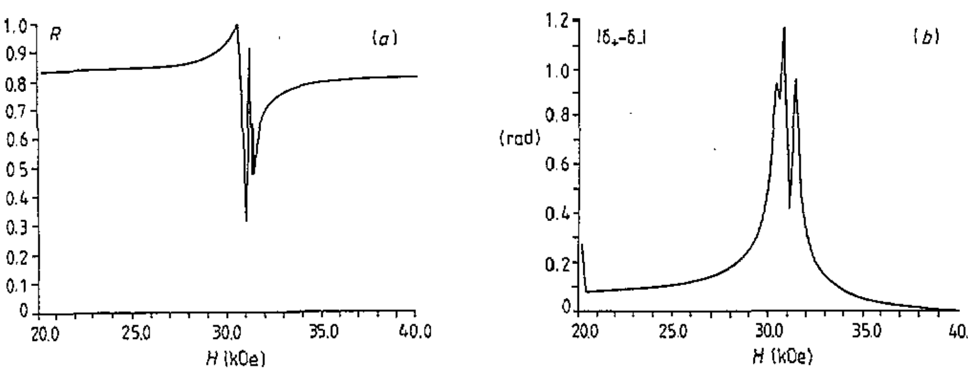

Figure 4. Variations in (*a*) R and (*b*) $|\delta_+ - \delta_-|$ with biasing field for a double magnetic layer separated by a dielectric layer (air). The parameters are $f = 90$ GHz, $4\pi M_1 = 3$ kOe, $4\pi M_2 = 1.75$ kOe, $n_f = 3.16$, $q_i = 45°$, and $d_1 = d_2 = 100$ μm. The separation d of the two magnetic films is 100 μm.

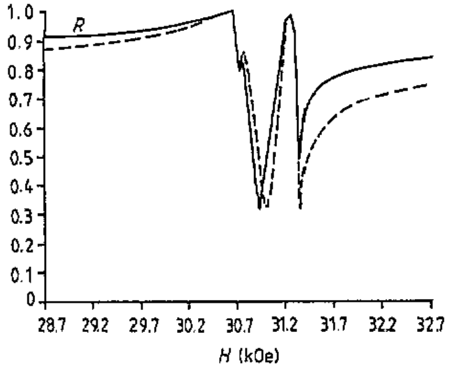

Figure 5. The influence of the separation of the two magnetic films on the reflection: ——, $d = 0$; – – –, $d = 100\ \mu$m. The other parameters are the same as in figure 4.

4. Magnetic polaritons of multilayers

Surface waves and bulk waves propagating along the surfaces and interfaces of a finite layer system correspond to

$$Y_0 + Y_{\text{eff}} = 0 \tag{30}$$

where Y_{eff} is the conductance of the magnetic layer system. Combining equation (19) with equation (30), we have

$$Y_0 Y_d P_{12} + Y_0 P_{11} + Y_d P_{22} + P_{21} = 0 \tag{31}$$

where $Y_0 = N_0 \cos(q_i)$ and $Y_d = N_d \cos(q_d)$. Through some manipulations, equation (31) can be written as follows:

$$\alpha_0 \alpha_d P_{12} + \alpha_0 P_{11} + \alpha_d P_{22} + P_{21} = 0 \tag{32}$$

where α_0 and α_d are the decay constants of surface waves in upper and lower non-magnetic media:

$$\alpha_0 = (\beta^2 - \omega^2 N_0/C^2)^{1/2} \qquad \alpha_d = (\beta^2 - \omega^2 N_d/C^2)^{1/2} \tag{33}$$

with $\mathbf{P} = \Pi_{i=1}^{N} \mathbf{P}_i$; β is the propagation wavenumber along the interfaces. Now

$$\mathbf{P}_i = \begin{bmatrix} \cosh(\alpha_f d_i) - [\beta u_2 \sinh(\alpha_f d)]/\alpha_f u_1 & [u_{\text{eff}} \sinh(\alpha_f d)]/\alpha_f \\ [(\alpha_f^2 - \beta^2 u_2^2/u_1^2) \sinh(\alpha_f d)]/u_{\text{eff}}\alpha_f & \cosh(\alpha_f d_i) + [\beta u_2 \sinh(\alpha_f d)]/\alpha_f u_1 \end{bmatrix}$$

$$= \begin{bmatrix} \chi_- \cosh(\alpha_f d_i) & \sinh(\alpha_f d_i)/\gamma \\ \gamma_v \sinh(\alpha_f d_i) & \chi_+ \cosh(\alpha_f d_i) \end{bmatrix} \tag{34}$$

$$\chi_- = 1 - [\beta u_2 \tanh(\alpha_f d_i)]/\alpha_f u_1 \qquad \chi_+ = 1 + [\beta u_2 \tanh(\alpha_f d_i)]/\alpha_f u_1$$

$$\gamma_v = (\alpha_f^2 - \beta^2 u_2^2/u_1^2)/u_{\text{eff}}\alpha_f \qquad \gamma = \alpha_f/u_{\text{eff}}.$$

The above equations (32) and (33) can be easily solved by a personal computer for any given finite magnetic layer system.

4.1. Examples

4.1.1. Semi-infinite magnetic medium. This case has been discussed by Hartstein *et al* [17] for a ferromagnetic medium and by Camley [18] for an antiferromagnetic medium.

The dispersion relation should correspond to

$$Y_0 + Y_{\text{eff}} = 0$$

with $Y_{\text{eff}} = L + M$; therefore

$$\alpha_0 + \alpha_f/u_{\text{eff}} + \beta u_2/u_1 u_{\text{eff}} = 0 \tag{35}$$

where $\alpha_f = (\beta^2 - \omega^2 u_{\text{eff}} N_f/C^2)^{1/2}$; u_1, u_2 and u_{eff} are different for a ferromagnetic medium and an antiferromagnetic film.

4.1.2. Symmetric single magnetic film. From equation (32), we have

$$(P_{11} + P_{22})\alpha_0 + P_{12}\alpha_0^2 + P_{21} = 0 \tag{36}$$

which reduces to

$$2u_{\text{eff}}\alpha_f\alpha_0/[-\alpha_f^2 + (\beta u_2/u_1)^2 - \alpha_0^2 u_{\text{eff}}^2] = \tanh(\alpha_f d_f). \tag{37}$$

Equation (37) is the same as that obtained by Karsono and Tilley [19].

4.1.3. Double magnetic film. For a non-magnetic medium, the matrix **P** takes the simple form

$$\mathbf{P} = \begin{bmatrix} \cosh(\alpha d) & \sinh(\alpha d)/\alpha \\ \alpha \sinh(\alpha d) & \cosh(\alpha d) \end{bmatrix} \tag{38}$$

with $\alpha = (\beta^2 - \omega^2 N/C^2)^{1/2}$. The system consists of a double magnetic layer of magnetizations M_{s1} and M_{s2} and thicknesses d_1 and d_2, separated by a non-magnetic layer of thickness d_0. The system is magnetized parallel to the film plane. For the Voigt geometry, the dispersion relations have the following simple form:

$$\frac{P_1 Q_1 P_2 Q_2[1 - \exp(2\alpha_f d_2)][1 - \exp(2\alpha_f d_1)]}{[P_2 Q_1 \exp(2\alpha_f d_2) - P_1 Q_2][P_2 Q_1 \exp(2\alpha_f d_1) - P_1 Q_2]} = \exp(2\alpha_f d_0) \tag{39}$$

where

$P_1 = \alpha_f - \beta k/\mu - \alpha_d \mu_{\text{eff}}$ $\qquad Q_1 = \alpha_f + \beta k/\mu + \alpha_d \mu_{\text{eff}}$

$P_2 = \alpha_f - \beta k/\mu + \alpha_d \mu_{\text{eff}}$ $\qquad Q_2 = \alpha_f + \beta k/u - \alpha_d \mu_{\text{eff}}$

$\alpha_d^2 = \beta^2 - \omega^2 \epsilon_d \mu_0$ $\qquad \alpha_f^2 = \beta^2 - \omega^2 \epsilon_f \mu_0 \mu_{\text{eff}} \mu_r$

$\mu_{\text{eff}} = [(\Omega_H + 1)^2 - \Omega^2]/(\Omega_H^2 + \Omega_H - \Omega^2)$ $\qquad \mu = \Omega_H/(\Omega_H^2 - \Omega^2)$ $\qquad k = \Omega/(\Omega_H^2 - \Omega^2)$.

The magnetostatic analyses of Gruberg [9] have been extended to EM analyses including retardation.

5. Conclusions and discussion

(1) A method for analysing the transmission and reflection of EM waves through magnetic multilayers and superlattices is presented. The method is based on the effective conductance of layered media, which can be expressed explicitly by the propagation matrix.

(2) The numerical results show that the EM modes of a magnetic layer system are excited and manifested as sharp dips in the s-polarized reflection and the dispersion curves of magnetic polaritons can be measured by a method similar to the ATR method. From the expressions for the matrix **P**, we know that the reflection is non-reciprocal, i.e. R and T for an incidence angle θ differ from those for $-\theta$. The non-reciprocal propagation is contained in M in equation (15). When q_i changes from $+\theta$ to $-\theta$, M changes its sign. The Y_{eff}-values for layered media are different for positive θ and negative $-\theta$. Therefore, the reflection of an EM microwave from a magnetized ferrimagnetic film is phase-wise and amplitude-wise non-reciprocal for any asymmetric layer system.

(3) The general dispersion relations for magnetic polaritons in arbitrary magnetic layer systems are derived by the propagation matrix method. It is interesting to note that all these equations can be written in the form

$$\alpha_0 \alpha_d P_{12} + \alpha_0 P_{11} + \alpha_d P_{22} + P_{21} = 0$$

for finite magnetic multilayers composed of N (N arbitrary) materials.

Clearly there remain a number of interesting problems which should be addressed in the future. For instance, we have investigated the surface polaritons of arbitrary propagation in a magnetic film [4]. For general cases of arbitrary propagation direction, the general matrix **P** should be a 4×4 matrix. The solutions of R and T for EM waves through magnetic multilayers can be solved by a 4×4 matrix **P**; this is at present under investigation.

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